Theorem 3.5.1: Given a den $(h, p, q, r)$ and $j = (q, \ldots, q)$ with $g_j = \frac{p}{q}$, define a Catalanomial

$$H := H_l(R_q, R_q, (l_1, \ldots, l_{q-1}, l_q) \setminus \{l_i \mid i \neq j\})$$

The length $l = q - 1 + g_j$ where

- $R_q$ is $\{1, \ldots, q\}$, $g_j$ are distinct because of the partition of $\{1\}$ into intervals of length $q - 1$.
- $R_q = \{1, \ldots, q\}$ is not necessarily maximal.

Then the polynomial part of it is given by

$$H_{pol} (x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} \sum_{k=0}^{\infty} \frac{1}{q^k} \omega_G(x)^k$$

Corollary 3.5.2: Given a den $(h, \beta, \gamma, \delta)$ and $\xi = \xi(h, \beta, \gamma, \delta)$ be the Schifffman-Sijbseh element represented by the same Catalanomial $H$ in Theorem 3.5.1.

Then

$$\xi - 1 = \sum_{i=0}^{\infty} \frac{x^i}{i!} \sum_{k=0}^{\infty} \frac{1}{q^k} \omega_G(x)^k$$

Proof: By Prop. 3.3.2, $\omega_G(x)^k = H_{pol}(x)$.

Hence by Theorem 3.5.1, $\xi - 1 = \omega_G(x)^k$. \hfill \Box

Corollary 3.5.3: Both $\sum_{n=0}^{\infty} \frac{x^n}{n!} \sum_{k=0}^{\infty} \frac{1}{q^k} \omega_G(x)^k$ and $\sum_{n=0}^{\infty} \frac{x^n}{n!} \omega_G(x)^k$ are $q, t$-symmetric.

Proof: By Theorem 3.5.1, the Catalanomial $H$ constructed in Theorem 3.5.1 is $q, t$-symmetric (and hence $t$-symmetric) and $H_{pol}(x)$ is $q, t$-symmetric (and hence $t$-symmetric). \hfill \Box

Remark: If the den has no nest, then Theorem 3.5.1 implies $H_{pol} = \xi - 1 = 0$.

Examples on the next q pages.